

A TREATISE
ON
CONIC SECTIONS:

CONTAINING

AN ACCOUNT OF SOME OF THE MOST IMPORTANT MODERN
ALGEBRAIC AND GEOMETRIC METHODS.

BY
GEORGE SALMON, D.D., D.C.L., LL.D., F.R.S.,
REGIUS PROFESSOR OF DIVINITY IN THE UNIVERSITY OF DUBLIN.

SIXTH EDITION.

London:
LONGMANS, GREEN, AND CO.
1879.

CAMBRIDGE:
PRINTED BY W. METCALFE AND SON, TRINITY STREET.

CONTENTS.

[Junior readers will find all essential parts of the theory of Analytical Geometry included in Chapters I., II., V., VI., X., XI., XII., omitting the articles marked with asterisks.]

CHAPTER I.

THE POINT.

	PAGE
Des Cartes' Method of Coordinates	1
Distinction of Signs	2
Distance between two Points	3
Its sign	4
Coordinates of Point cutting that Distance in a given Ratio	5
Transformation of Coordinates	6
does not change Degree of an Equation	9
Polar Coordinates	10

CHAPTER II.

THE RIGHT LINE.

Two Equations represent Points	11
A single Equation represents a Locus	12
Geometric representation of Equations	13
Equation of a Right Line parallel to an Axis	14
through the Origin	15
in any Position	16
Meaning of the Constants in Equation of a Right Line	17
Equation of a Right Line in terms of its Intercepts on the Axes	18
in terms of the Perpendicular on it from Origin, and the Angles it makes with Axes	19
Expression for the Angles a Line makes with Axes	20
Angle between two Lines	21
Equation of Line joining two given Points	23
Condition that three Points shall be on one Right Line	24
Coordinates of Intersection of two Right Lines	25
Middle Points of Diagonals of a Quadrilateral are in a Right Line (see also p. 62)	26
Equation of Perpendicular on a given Line	26
of Perpendiculars of Triangle	27
of Perpendiculars at Middle Points of Sides	27
of Line making a given Angle with a given Line	27

	PAGE
Length of Perpendicular from a Point on a Line	28
Equations of Bisectors of Angles between two given Right Lines	29
Area of Triangle in terms of Coordinates of its Vertices	30
Area of any Polygon	31
Condition that three Lines may meet in a Point (see also p. 34)	32
Area of Triangle formed by three given Lines	32
Equation of Line through the Intersection of two given Lines	33
Test that three Equations may represent Right Lines meeting in a Point	34
Connexion between Ratios in which Sides of a Triangle are cut by any Transversal	35
by Lines through the Vertices which meet in a Point	36
Polar Equation of a Right Line	36

CHAPTER III.

EXAMPLES ON THE RIGHT LINE.

Investigation of Rectilinear Loci	39
of Loci leading to Equations of Higher Degree	47
Problems where it is proved that a Moveable Line always passes through a Fixed Point	47
Centre of Mean Position of a series of Points	50
Right Line passes through a Fixed Point if Constants in its Equation be connected by a Linear Relation	50
Loci solved by Polar Coordinates	51

CHAPTER IV.

THE RIGHT LINE.—ABRIDGED NOTATION.

Meaning of Constant k in Equation $\alpha = k\beta$	53
Bisectors of Angles, Bisectors of Sides, &c. of a Triangle meet in a Point	34, 54
Equations of a pair of Lines equally inclined to α, β	55
Theorem of Anharmonic Section proved	55
Algebraic Expression for Anharmonic Ratio of a Pencil	56
Homographic Systems of Lines	57
Expression of Equation of any Right Line in terms of three given ones	57
Harmonic Properties of a Quadrilateral proved (see also p. 317)	57
Homologous Triangles: Centre and Axis of Homology	59
Condition that two Lines should be mutually Perpendicular	59
Length of Perpendicular on a Line	60
Perpendiculars at middle Points of Sides meet in a Point	34, 60
Angle between two Lines	60
Trilinear Coordinates	61
Trilinear Equation of Parallel to a given Line	61
of Line joining two Points	62
Proof that middle Points of Diagonals of Quadrilateral lie in a Right Line	62
Intersections of Perpendiculars, of Bisectors of Sides, and of Perpendiculars at middle Points of Sides, lie in a Right Line	63
Equation of Line at infinity	64
Cartesian Equations a case of Trilinear	65
Tangential Coordinates	65
Reciprocal Theorems	66

CONTENTS.

v

CHAPTER V.

RIGHT LINES.

	PAGE
Meaning of an Equation resolvable into Factors	67
of a Homogeneous Equation of the n^{th} Degree	68
Imaginary Right Lines	69
Angle between two Lines given by a single Equation	70
Equation of Bisectors of Angles between these Lines	71
Condition that Equation of second Degree should represent Right Lines (see also pp. 149, 153, 155, 266)	72
Number of conditions that higher Equations may represent Right Lines	74
Number of terms in Equation of n^{th} Degree	74

CHAPTER VI.

THE CIRCLE.

Equation of Circle	75
Conditions that general Equation may represent a circle	75
Coordinates of Centre and Radius	76
Condition that two Circles may be concentric	77
that a Curve shall pass through the origin	77
Coordinates of Points where a given Line meets a given Circle	77
Imaginary Points	77
General definition of Tangents	78
Condition that Circle should touch either Axis	79
Equation of Tangent to a Circle at a given Point	80, 81
Condition that a Line should touch a Circle	81
Equation of Polar of a Point with regard to a Circle or Conic	82, 83
Length of Tangent to a Circle	84
Line cut harmonically by a Circle, Point, and its Polar	85
Equation of pair of Tangents from a given Point to a Circle	85
Circle through three Points (see also p. 130)	86
Condition that four Points should lie on a Circle, and its Geometrical meaning	86
Polar Equation of a Circle	87

CHAPTER VII.

EXAMPLES ON CIRCLE.

Circular Loci	88
Condition that intercept by Circle on a given Line may subtend a right Angle at a given Point	90
If a Point A lie on the polar of B , B lies on the polar of A	91
Conjugate and self-conjugate Triangles	91
Conjugate Triangles Homologous	92
If two Chords meet in a Point, Lines joining their extremities transversely meet on its Polar	92
Distances of two Points from the centre, proportional to the distance of each from Polar of other	93
Expression of Coordinates of Point on Circle by auxiliary Angle	94
Problems where a variable Line always touches a Circle	95
Examples on Circle solved by Polar Coordinates	96