

A  
A TREATISE  
ON  
CONIC SECTIONS:

CONTAINING

AN ACCOUNT OF SOME OF THE MOST IMPORTANT MODERN  
ALGEBRAIC AND GEOMETRIC METHODS.

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[Junior readers will find all essential parts of the theory of Analytical Geometry included in Chapters I., II., V., VI., X., XI., XII., omitting the articles marked with asterisks.]

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