

A TREATISE
ON THE
ANALYTIC GEOMETRY
OF
THREE DIMENSIONS.

BY
GEORGE SALMON, D.D.,
FELLOW AND TUTOR OF TRINITY COLLEGE, DUBLIN.

Dublin:
HODGES, SMITH, & CO., GRAFTON STREET,
BOOKSELLERS TO THE UNIVERSITY.
1862.

CAMBRIDGE:

PRINTED BY WILLIAM METCALFE, GREEN STREET.

PREFACE.

IN writing a preface, what I am most tempted to do is to enumerate and account for the omissions of this treatise; if it were not that the size to which the volume has swelled, renders it needless for me to apologize for not having made it larger. It may be right however to mention that the chapters of this work were written and sent to press at intervals as I found leisure, and that the earlier part of the book has been in type more than a year. This will explain why no use has been made of some recent works and memoirs. In particular, I must express my regret that Hesse's "Lectures on the Analytic Geometry of Space" came too late to be of service to me.

In treating of the less modern parts of the Science, I have usually had Leroy's and Gregory's Treatises before me. The parts of this work which correspond to the contents of theirs are, the Theory of Surfaces of the Second Order, pp. 1—88; of the Curvature of Surfaces, pp. 197—223; of what I have called the Non-Projective Properties of Curves of Double Curvature, pp. 259—277 and of Families

of Surfaces, pp. 312—338. Junior readers will probably find all the information they require, if to the course here marked out they add part of the Theory of Confocal Surfaces, pp. 129—138, and the General Theory of Surfaces, Chap. x.

I have to acknowledge with thanks the kind readiness with which assistance was afforded me by any of my friends whose help I claimed. Those to whom I am most indebted are Dr. Hart and the Messrs. Roberts; but I have received occasional assistance from Messrs. Townsend, Williamson, and Gray, to the latter of whom I owe the list of Errata which follows the Table of Contents.

I have to thank the Board of Trinity College, for their liberality in contributing to the expense of publication.

TRINITY COLLEGE, DUBLIN,
May, 1862.

CONTENTS.

—o—

CHAPTER I.

THE POINT.

	PAGE
Method of Co-ordinates	1
Properties of Projections	3
Co-ordinates of point cutting in a given ratio the distance between two points	5
Co-ordinates of centre of a tetrahedron	6
Distance between two points (rectangular Co-ordinates)	6
Direction-cosines of a line	7
Area of a figure in terms of areas of its projections	7
Angle between two lines in terms of their direction-cosines	8
Perpendicular distance of a point from a line	8
Direction-cosines of the perpendicular to the plane of two lines	9
TRANSFORMATION OF CO-ORDINATES	9
Distance between two points (oblique Co-ordinates)	11
Degree of an equation unaltered by transformation	11

CHAPTER II.

INTERPRETATION OF EQUATIONS.

Meaning of a single equation; of a system of two or three equations	12
Every plane section of a surface of the n^{th} degree is a curve of the n^{th} degree	14
Every right line meets a surface of the n^{th} degree in n points	14
Order of a curve in space defined	14
Three surfaces of degrees m, n, p , intersect in mnp points	15
Cylindrical surfaces defined	15

CHAPTER III.

THE PLANE.

Every equation of the first degree represents a plane	16
Equation of a plane in terms of its direction-cosines and perpendicular from origin	16
Angle between two planes	17
Condition that two planes may be mutually perpendicular	17
Equation of plane in terms of intercepts made on axes	17