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A TREATISE
ON
ORDINARY AND PARTIAL
DIFFERENTIAL EQUATIONS

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PREFACE.

THE treatment of the subject of Differential Equations here presented will, it is hoped, be found complete in all those portions which bear upon their practical applications, and in the discussion of their theory so far as it can be adequately treated without the use of the complex variable. The topics included and the order pursued are sufficiently indicated by the table of contents.

An amount of space somewhat greater than usual has been devoted to the geometrical illustrations which arise when the variables are regarded as the rectangular coordinates of a point. This has been done in the belief that the conceptions peculiar to the subject are more readily grasped when embodied in their geometric representations. In this connection the subject of singular solutions of ordinary differential equations and the conception of the characteristic in partial differential equations may be particularly mentioned.

Particular attention has been paid to the development of symbolic methods, especially in connection with the operator $x \frac{d}{dx}$, for which, in accordance with recent usage, the symbol ϕ has been adopted. Some new applications of this symbol have been made.