ELEMENTS OF ALGEBRA

PRELIMINARY TO THE DIFFERENTIAL CALCULUS

AND FIT FOR THE HIGHER CLASSES OF SCHOOLS IN WHICH

THE PRINCIPLES OF ARITHMETIC ARE TAUGHT.

BY

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SECOND EDITION.

"What a benefite that onely thyng is, to haue the witte whetted and sharpened, I neade not trauell to declare, sith all men confesse it to be as greate as maie be. Excepte any witlesse persone thinke he maie bee to wise. But he that moste feareth that, is leaste in daunger of it. Wherefore to conclude, I see moare menne to acknowledge the benefite of nomber, than I can espie willyng to studie, to attaine the benefites of it. Many praise it, but fewe dooe greatly practise it: onlesse it bee for the vulgare practice, concernyng Merchaundes trade. Wherein the desire and hope of gain, maketh many willyng to sustaine some trauell. For aide of whom, I did sette forth the firste parte of *Arithmetike*. But if thei knewe how farre this seconde parte, dooeth excell the firste parte, thei would not accoumpte any tyme loste, that were imploied in it. Yea thei would not thinke any tyme well bestowed, till thei had gotten soche habilitie by it, that it might be their aide in al other studies."—ROBERT RECORDE.

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PREFACE

TO THE

FIRST EDITION.

In the title-page I have endeavoured to make it clear that it will be impossible to teach algebra on the usual plan by means of this work. It is intended only for such students as *have that sort of knowledge of the principles of arithmetic which comes by demonstration*, and whose reasoning faculties have therefore already undergone some training.

Algebra, as an art, can be of no use to any one in the business of life; certainly not as taught in schools. I appeal to every man who has been through the school routine whether this be not the case. Taught as an art it is of little use in the higher mathematics, as those are made to feel who attempt to study the differential calculus without knowing more of its principles than is contained in books of rules.

The *science* of algebra, independently of any of its uses, has all the advantages which belong to mathematics in general as an object of study, and which it is not necessary to enumerate. Viewed either as a science of quantity, or as a language of symbols, it may be made of the greatest service to those who are sufficiently acquainted with arithmetic, and have sufficient power of comprehension, to enter fairly upon its difficulties. But if, to meet the argument that boys cannot learn algebra in its widest form, it be proposed to evade the real and efficient part of the science, whether by presenting results only in the form of rules, or by omitting and taking for granted what should be inserted and proved,

for the purpose of making it appear that something *called* algebra has been learned: I reply, that it is by no means necessary, except for show, that the word algebra should find a place in the list of studies of a school; that, after all, the only question is, whether what is taught under that name be worth the learning; and that if real *algebra*, such as will be at once an exercise of reasoning, and a useful preliminary to subsequent studies, be too difficult, it must be deferred. Of this I am quite sure, that the student who has no more knowledge of arithmetic—that is, of the reasoning on which arithmetical notation and processes are built—than usually falls to the lot of those who begin algebra at school—that is, I believe, begin to add *positive* and *negative* quantities together,—will sooner find his way barefoot to Jerusalem than understand the greater part of this work. And I may say the same of every work on algebra, containing reasoning and not rules, which I have ever seen; provided it contained any of the branches of the subject which are of most usual application in the higher parts of mathematics.

The special object to which this work is devoted is the developement of such parts of algebra as are absolutely requisite for the study of the differential calculus, the most important of all its applications. The former science is now so extensive, that some particular line must be marked out by every writer of a small treatise. The very great difficulty of the differential calculus has always been a subject of complaint; and it has frequently been observed that no one knows exactly what he is doing in that science until he has made considerable progress in the mechanism of its operations. I have long believed the reason of this to be that the fundamental notions of the differential calculus are conventionally, and with difficulty, excluded from algebra, in which I think they ought to occupy an early and prominent place. I have, therefore, without any attention to the agreement by which the theory of limits is never suffered to make

its appearance in form until the commencement of the differential calculus, introduced limits throughout my work: and I can certainly assure the student, that, though I have perhaps thereby increased the difficulty of the subject, the additional quantity of thought and trouble is but a small dividend upon that which he would afterwards have had to encounter, if he had been permitted to defer the considerations alluded to till a later period of his mathematical course. On those who offer theoretical objections to the introduction of limits in a work on algebra lies the *onus* of shewing that they are not already introduced, even in arithmetic. What is $\sqrt{2}$, supposing geometry and limits both excluded?

I have been sparing of examples for practice in the earlier part of the work, and this because I have always found that manufactured instances do not resemble the combinations which actually occur. They are but a sort of parade exercise, which cannot be made to include the means of meeting the thousand contingencies of actual service. The only method of furnishing useful cases is to take some inverse process, and the verification of literal equations (as in the seventh and following pages of this work) is the most obvious. With these the student can furnish himself at pleasure, the test of correctness being the ultimate agreement of the two sides, after the value of the unknown quantity has been substituted.

The only remaining caution which he will need is, not to proceed too quickly, especially in the earlier part of the work. He must remember that he is engaged upon a very difficult subject, and that if he does not find it so, it is, most probably, because he does not understand what he is about. Wherever an instance or a process occurs, he should take others as like them as he can, and assure himself, by the reasoning in the work, that he has obtained a true result.

It was at first my intention to write a second volume on the higher parts of the subject. But, considering that of