

A TREATISE
ON
ELEMENTARY GEOMETRY

WITH APPENDICES CONTAINING
A COLLECTION OF EXERCISES FOR STUDENTS
AND
AN INTRODUCTION TO MODERN GEOMETRY.

BY
WILLIAM CHAUVENET, LL.D.,
PROFESSOR OF MATHEMATICS AND ASTRONOMY IN WASHINGTON UNIVERSITY.

PHILADELPHIA
J. B. LIPPINCOTT & CO.
1871.

Entered according to Act of Congress, in the year 1870, by

J. B. LIPPINCOTT & CO.,

In the Clerk's Office of the District Court of the United States for the Eastern District of
Pennsylvania.

LIPPINCOTT'S PRESS,
PHILADELPHIA.

P R E F A C E.

THE invention of Analytic Geometry by DESCARTES in the early part of the seventeenth century, quickly followed by that of the Infinitesimal Calculus by NEWTON and LEIBNITZ, produced a complete revolution in the mathematical sciences themselves and accelerated in an astonishing degree the progress of all the sciences in which mathematics are applied, but arrested for a time the progress of pure geometry. The new methods, characterized by great generality and facility in their application to problems of the most varied kinds, offered to the succeeding generations of investigators more inviting fields of research and promises of surer and richer reward than the special and apparently more restricted methods of the ancients. During the eighteenth century hardly any important addition to geometry was made that was not the direct product, either of the Cartesian method alone, or of that method in alliance with the Infinitesimal Calculus.

With the present century, however, a new era commenced in pure geometry. The first impulse was given by the Descriptive Geometry of MONGE; then followed CARNOT's Theory of Transversals, PONCELET's Projective Properties of Figures and Method of Reciprocal Polars, the researches of STEINER, POINSOT, GERGONNE, CAYLEY, MACCULLAGH, and many others, crowned by the brilliant discoveries of CHASLES.

All this progress, it is true, has been chiefly in the higher departments of pure geometry, and has not yet essentially changed

the substance or form of what is known as Elementary Geometry, which is little more than the Geometry of EUCLID in a modern dress, with certain necessary additions in solid geometry; for, although some of the recent discoveries are of a remarkably simple character and (if simplicity were the only requisite) might be introduced into the elements, it is generally conceded that in elementary instruction it is most expedient to commence with the Euclidian geometry, and to reserve the new developments for subsequent study under the name of the *Modern Geometry*.

Nevertheless, this advance in the general science has not failed to produce its legitimate effect upon the primary branch; and the modern treatises on the elements, especially in France, from that of LEGENDRE in 1794 to that of ROUCHÉ and COMBEROUSSE in 1868, exhibit a gradual and marked improvement both in matter and method.

In the following treatise, designed especially for use in colleges and schools, I have endeavored to set forth the elements with all the rigor and completeness demanded by the present state of the general science, without seriously departing from the established order of the propositions, or sacrificing the simplicity of demonstration required in a purely elementary work. Some subjects, not usually included in elementary works, are so placed that they may be omitted without breaking the chain of demonstration, and the remainder may be used as an abridged course in those schools where the time allotted to the study does not suffice for the perusal of the whole. Such, for example, are the articles on Maxima and Minima at the end of Book V. and those on Similar Polyedrons and the Regular Polyedrons at the end of Book VII.

As the student can make no solid acquisitions in geometry without frequent practice in the application of the principles he has acquired, a copious collection of exercises is given in the Appendix. The discouraging difficulties which the young student commonly experiences in his first attempts at demonstrating new theorems, or solving new problems, are here obviated in a great

degree by giving him such suggestions for the solution of many of the exercises as may fairly be presumed to be necessary for him at the successive stages of his progress. These suggestions are given with less and less frequency as he advances, and he is finally left to rely entirely upon his own resources when he may be supposed to have acquired by practice considerable familiarity with principles, and dexterity in their application.

The Appendix on the *Modern Geometry*, although restricted to the properties of the straight line and circle, will serve a good purpose, it is hoped, either as an introduction to such works as those of PONCELET and CHASLES in which the methods of pure geometry are employed, or as a companion to the works of SALMON and others in which the new geometry is treated by the analytic method.

In the preparation of this work, I have derived valuable aid from a number of the more recent French treatises on Elementary Geometry, and especially from those of BOBILLIER, BRIOT, COMPAGNON, LEGENDRE (edited by Blanchet), and the very complete *Traité de Géométrie Élémentaire* of ROUCHÉ and COMBEROUSSE. The last named work has furnished many of the exercises of Appendix I. and much of the matter of Appendix II.

WASHINGTON UNIVERSITY,
St. LOUIS, June 1, 1869.