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СОДЕРЖАНИЕ

Abanina D. A. On Borel's extension theorem for general Beurling classes of ultradifferentiable functions	3
Buskes G., Kusraev A. G. Representation and extension of orthoregular bilinear operators	16
Danchev P. V. A note on weakly \aleph_1 -separable p -groups	30
Grabarnik G. Ya., Katz A. A., Shwartz L. On non-commutative ergodic type theorems for free finitely generated semigroups	38
Коробейник Ю. Ф. О нулях одного класса гармонических функций	48
Teamah A. A. M., Bakouch H. S. Some asymptotic properties of a kernel spectrum estimate with different multitapers	56
Тюриков Е. В. Об одной граничной задаче теории бесконечно малых изгибаний поверхности	62

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ON BOREL'S EXTENSION THEOREM FOR GENERAL BEURLING CLASSES OF ULTRADIFFERENTIABLE FUNCTIONS

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We obtain necessary and sufficient conditions under which general Beurling class of ultradifferentiable functions admits a version of Borel's extension theorem.

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1. Introduction

DEFINITION 1.1. An increasing continuous function $\omega : [0, \infty) \rightarrow [0, \infty)$ is called a weight function if

$$\begin{aligned} \log t &= o(\omega(t)), \quad t \rightarrow \infty; \\ \omega(t) &= O(t); \quad t \rightarrow \infty; \\ \varphi_\omega(x) &:= \omega(e^x) \text{ is convex on } [x_0, \infty). \end{aligned}$$

A weight function ω with $\int_1^\infty t^{-2} \omega(t) dt < \infty$ is called nonquasianalytic.

Denote by W_\uparrow the set of all sequences $\Omega = \{\omega_n\}_{n=1}^\infty$ of weight functions with the following property: for each $n \in \mathbb{N}$ there exists a $C_n > 0$ such that

$$\omega_n(t) + \log(t+1) \leq \omega_{n+1}(t) + C_n \quad \text{for } t \geq 0. \quad (1)$$

By W_\uparrow^{nq} denote the set of all sequences $\Omega = \{\omega_n\}_{n=1}^\infty$ of nonquasianalytic weight functions ω_n . Without loss of generality we can assume that

$$\omega_n(t) \leq \omega_{n+1}(t) \quad \text{for } t \geq 0 \text{ and } n \in \mathbb{N}.$$

The Young conjugate $\varphi_\omega^* : [0, \infty) \rightarrow [0, \infty)$ of φ_ω is defined by

$$\varphi_\omega^*(y) := \sup\{xy - \varphi_\omega(x) : x \geq 0\}.$$

For $A \in (0, \infty)$ we define the space

$$\mathcal{E}_\omega(\Pi_A^N) := \left\{ f \in C^\infty(\Pi_A^N) : |f|_{\omega, A, N} := \sup_{\alpha \in \mathbb{N}_0^N} \sup_{\|x\| \leq A} \frac{|f^{(\alpha)}(x)|}{e^{\varphi_\omega^*(|\alpha|)}} < \infty \right\},$$

where $\Pi_A^N := \{x \in \mathbb{R}^N : \|x\| \leq A\}$, $\|x\| := \max\{|x_j| : 1 \leq j \leq N\}$ for $x = (x_1, \dots, x_N) \in \mathbb{R}^N$, $|\alpha| := \alpha_1 + \dots + \alpha_N$ for $\alpha = (\alpha_1, \dots, \alpha_N) \in \mathbb{N}_0^N$, $f^{(\alpha)} := \frac{\partial^{|\alpha|} f}{\partial x_1^{\alpha_1} \dots \partial x_N^{\alpha_N}}$.