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ON BOREL'S EXTENSION THEOREM FOR GENERAL BEURLING CLASSES OF ULTRADIFFERENTIABLE FUNCTIONS

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We obtain necessary and sufficient conditions under which general Beurling class of ultradifferentiable functions admits a version of Borel's extension theorem.

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1. Introduction

Definition 1.1. An increasing continuous function $\omega : [0,\infty) \to [0,\infty)$ is called a weight function if

$$\begin{aligned} &\log t = o(\omega(t)),\ t \to \infty;\\ &\omega(t) = O(t);\ t \to \infty;\\ &\varphi_\omega(x) := \omega(e^x) \text{ is convex on } [x_0,\infty). \end{aligned}$$

A weight function ω with $\int\limits_1^\infty t^{-2}\omega(t)\,dt < \infty$ is called nonquasianalytic.

Denote by $W_\uparrow$ the set of all sequences $\Omega = \{\omega_n\}_{n=1}^\infty$ of weight functions with the folllowing property: for each $n \in \mathbb{N}$ there exists a $C_n > 0$ such that

$$\omega_n(t) + \log(t+1) \leqslant \omega_{n+1}(t) + C_n \quad \text{for } t \geqslant 0. \tag{1}$$

By $W_\uparrow^{nq}$ denote the set of all sequences $\Omega = \{\omega_n\}_{n=1}^\infty$ of nonquasianalytic weight functions ω_n. Without loss of generality we can assume that

$$\omega_n(t) \leqslant \omega_{n+1}(t) \quad \text{for } t \geqslant 0 \text{ and } n \in \mathbb{N}.$$

The Young conjugate $\varphi_\omega^* : [0,\infty) \to [0,\infty)$ of φ_ω is defined by

$$\varphi_\omega^*(y) := \sup\{xy - \varphi_\omega(x) : x \geqslant 0\}.$$

For $A \in (0,\infty)$ we define the space

$$\mathscr{E}_\omega(\Pi_A^N) := \left\{ f \in C^\infty(\Pi_A^N) : \ |f|_{\omega,A,N} := \sup_{\alpha\in\mathbb{N}_0^N} \sup_{\|x\|\leqslant A} \frac{\left|f^{(\alpha)}(x)\right|}{e^{\varphi_\omega^*(|\alpha|)}} < \infty \right\},$$

where $\Pi_A^N := \{x \in \mathbb{R}^N : \|x\| \leqslant A\}$, $\|x\| := \max\{|x_j| : 1 \leqslant j \leqslant N\}$ for $x = (x_1,\dots,x_N) \in \mathbb{R}^N$, $|\alpha| := \alpha_1 + \ldots + \alpha_N$ for $\alpha = (\alpha_1,\dots,\alpha_N) \in \mathbb{N}_0^N$, $f^{(\alpha)} := \dfrac{\partial^{|\alpha|} f}{\partial x_1^{\alpha_1} \ldots \partial x_N^{\alpha_N}}$.