РОССИЙСКАЯ АКАДЕМИЯ НАУК
ВЛАДИКАВКАЗСКИЙ НАУЧНЫЙ ЦЕНТР
ИНСТИТУТ ПРИКЛАДНОЙ МАТЕМАТИКИ И ИНФОРМАТИКИ

ВЛАДИКАВКАЗСКИЙ МАТЕМАТИЧЕСКИЙ ЖУРНАЛ

Том 9, Выпуск 2 — апрель–июнь, 2007

СОДЕРЖАНИЕ

Владикавказ
2007

Владикавказский математический журнал
апрель–июнь, 2007, Том 9, Выпуск 2

UDC 512.742

ON A DECOMPOSITION EQUALITY IN MODULAR GROUP RINGS

P. V. Danchev

Let G be an abelian group such that $A \leqslant G$ with p-component A_p and $B \leqslant G$, and let R be a commutative ring with 1 of prime characteristic p with nil-radical $N(R)$. It is proved that if $A_p \nsubseteq B_p$ or $N(R) \neq 0$, then $S(RG) = S(RA)(1 + I_p(RG; B)) \iff G = AB$ and $G_p = A_pB_p$. In particular, if $A_p \neq 1$ or $N(R) \neq 0$, then $S(RG) = S(RA) \times (1 + I_p(RG; B)) \iff G = A \times B$. So, the question concerning the validity of this formula is completely exhausted. The main statement encompasses both the results of this type established by the author in (Hokkaido Math. J., 2000) and (Miskolc Math. Notes, 2005). We also point out and eliminate in a concrete situation an error in the proof of a statement due to T. Zh. Mollov on a decomposition formula in commutative modular group rings (Proceedings of the Plovdiv University-Math., 1973).

Mathematics Subject Classification (2000): 16S34, 16U60, 20K10, 20K21.

Key words: direct factors, decompositions, normed unit groups, homomorphisms.

1. Introduction

Traditionally, suppose RG is the group ring (often regarded as an R-algebra) of an abelian group G over a commutative ring with identity of prime characteristic, for instance, p. As usual, $V(RG)$ denotes the group of normalized units in RG, and $S(RG)$ is its Sylow p-component. For any subgroup C of G, the symbol $I(RG; C)$ will designate the relative augmentation ideal of RG with respect to C, and $I_p(RG; C)$ designates its nil-radical.

For an abelian group G, the letter G_p will denote its p-torsion part and for a commutative unitary ring R, the letter $N(R)$ denotes its nil-radical.

In a series of our investigations (e. g. [1–4, 6, 7]), we study how the direct decomposition of G can be translated on $S(RG)$; another treatment but of G_p was demonstrated in [5].

In [9] we generalized the foregoing results of this direction by considering an ordinary, not necessarily direct, decomposition and by exploring how such a decomposing of $S(RG)$ implies a corresponding one of G and G_p.

The aim of this article is to strengthen the aforementioned results from [9] and to settle completely the existence of such a decomposition formula for $S(RG)$. In doing that, we use some helpful facts that are of independent interest as well.

In closing, we correct a proof by Mollov [11] in a rather special case. The generalized version is wide-open yet. Likewise, we demonstrate that some assertions from [13] are not original and can be simplified in a more convenient form not as they stand.